

## 8.1 - I will use geometric mean to solve problems

### real life example

Photographing very tall or very wide objects can be challenging. It can be difficult to include the entire object in your shot without distorting the image. If your camera is set for a vertical viewing angle of  $90^\circ$  and you know the height of the object you wish to photograph, you can use the geometric mean of the distance from the top of the object to your camera level and the distance from the bottom of the object to camera level.



when the means of a proportion are the same number, that number is called the **geometric mean**

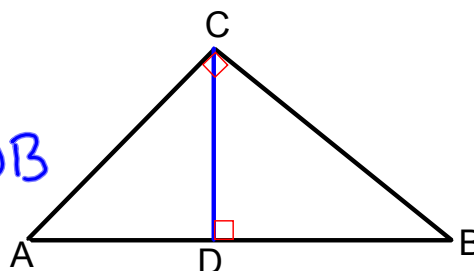
**geometric mean** - the geometric mean of two positive numbers  $a$  and  $b$  is the number  $x$  such that  $a/x = x/b$ . So  $x^2 = ab$  and  $x = \sqrt{ab}$

$$\frac{a}{x} = \frac{x}{b} \quad \sqrt{x^2} = \sqrt{ab} \quad x = \sqrt{ab}$$

Theorem 8.1 -

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

$$\triangle ACB \sim \triangle ADC \sim \triangle CDB$$

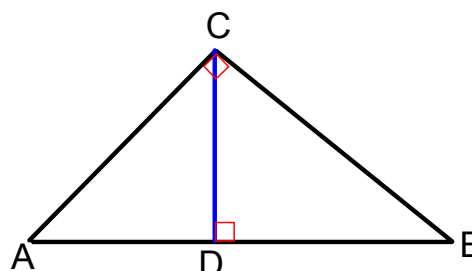


## Theorem 8.2 -

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of the altitude is the geometric mean of the lengths of the two segments.

$$\frac{CD}{AD} = \frac{DB}{CD}$$



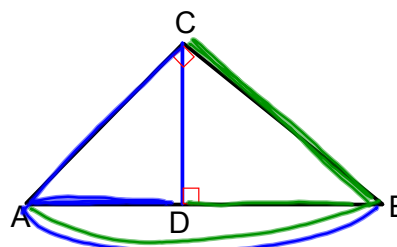
## Theorem 8.3 -

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments.

The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

$$\frac{CA}{AB} = \frac{AD}{CA}$$

$$\frac{CB}{AB} = \frac{DB}{CB}$$



Find the geometric mean between 8 and 10.

$$\frac{8}{x} \times \frac{x}{10} \quad \sqrt{x^2} = \sqrt{80} < \frac{8+10}{2}$$

$$x = \sqrt{2 \cdot 5}$$

$$x = 4\sqrt{5}$$

5 and 45

$$\frac{5}{x} \times \frac{x}{45}$$

$$\sqrt{x^2} = \sqrt{225}$$

$$x = 15$$

12 and 15

$$\frac{12}{x} \times \frac{x}{15}$$

$$x^2 = \sqrt{12 \cdot 15} = 6\sqrt{5}$$

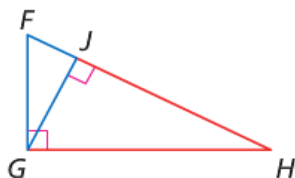
40 and 15

$$\frac{x}{40} \times \frac{15}{x}$$

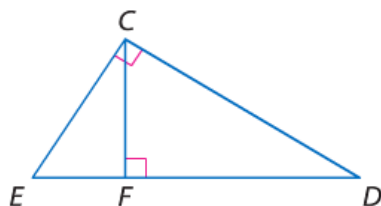
$$\sqrt{x^2} = \sqrt{600}$$

$$x = 10\sqrt{6}$$

Write a similarity statement identifying the three similar right triangles in the figure. (thm 8.1)



$$\triangle FGH \sim \triangle FJG \sim \triangle GJH$$



$$\triangle CED \sim \triangle CFE \sim \triangle CFD$$

Find  $x$ ,  $y$ , and  $z$ .

**Thm 8.2**  
 $\frac{x}{5} = \frac{20}{x}$   
 $x^2 = 100$   
 $x = 10$

**Thm 8.3**  
 $\frac{y}{25} = \frac{5}{y}$   
 $y^2 = 125$   
 $y = 5\sqrt{5}$

**Thm 8.2**  
 $\frac{z}{25} = \frac{20}{z}$   
 $z^2 = 25 \cdot 20$   
 $z = 10\sqrt{5}$

**Thm 8.3**  
 $\frac{x}{33} = \frac{8}{x}$   
 $x^2 = 8 \cdot 33$   
 $x = 2\sqrt{66}$

**Thm 8.3**  
 $\frac{y}{33} = \frac{25}{y}$   
 $y^2 = 25 \cdot 33$   
 $y = 5\sqrt{33}$

**Thm 8.2**  
 $\frac{z}{8} = \frac{25}{z}$   
 $z^2 = 8 \cdot 25$   
 $z = 10\sqrt{2}$

**Thm 8.2**  
 $\frac{12}{9} = \frac{x}{12}$   
 $9x = 144$   
 $x = 16$

**Thm 8.2**  
 $\frac{y}{25} = \frac{16}{y}$   
 $y^2 = 25 \cdot 16$   
 $y = 20$

**Thm 8.2**  
 $\frac{z}{25} = \frac{9}{z}$   
 $z^2 = 9 \cdot 25$   
 $z = 15$

$\frac{16}{x} = \frac{8}{16}$   
 $8x = 256$   
 $x = 32$

$\frac{z}{8} = \frac{40}{z}$   
 $z^2 = 320$   
 $z = 8\sqrt{5}$

$\frac{y}{40} = \frac{32}{y}$   
 $y^2 = 1280$   
 $y = 16\sqrt{5}$

$\frac{x}{3} = \frac{12}{x}$   
 $x^2 = 36$   
 $x = 6$

$\frac{y}{3} = \frac{15}{y}$   
 $y^2 = 45$   
 $y = 3\sqrt{5}$

$\frac{z}{12} = \frac{15}{z}$   
 $z^2 = 180$   
 $z = 6\sqrt{5}$

pg. 541 #~~8-22~~ even, ~~30, 32, 36~~

8, 10, 14, 18-22 even, 36